

University of Sussex

QED corrections for precision experiments

Yannick Ulrich

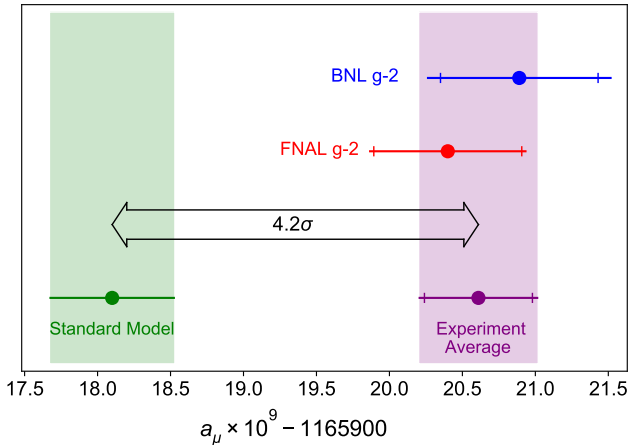
IPPP, University of Durham

28TH FEBRUARY 2022

I hope to address the following

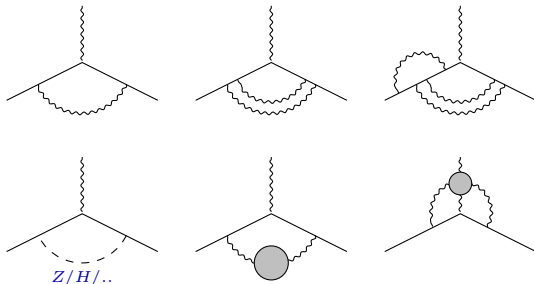
- $\alpha_{\text{QED}} \ll 1$, so why bother?
- ⇒ where do QED corrections matter?
- what challenges?
 - how to solve them (in pictures!)
 - some phenomenology (more pretty pictures!)
 - vision of the future

most precise measurement of $g - 2$

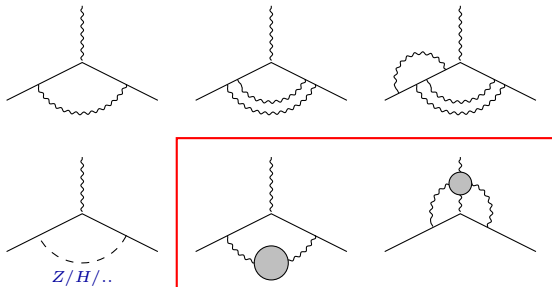


⇒ needs precise theory

many Feynman diagrams, incl. non-perturbative

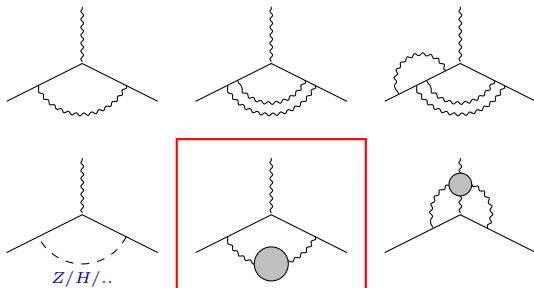


many Feynman diagrams, incl. non-perturbative



leading theory uncertainty

many Feynman diagrams, incl. non-perturbative

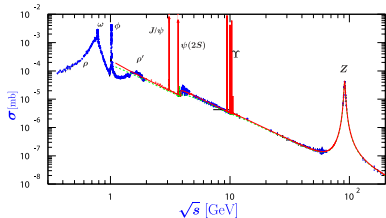


the hadronic vacuum polarisation

using optical theorem $s > 0$

$$\int ds \left(K(s) \right) \left(\text{Diagram: two lines merging into a wavy line which then splits into two lines hitting a grey semi-circle} \right)$$

\Rightarrow very messy!



using optical theorem $s > 0$

$$\int ds \left(K(s) \right) \left(\text{Diagram: s-channel exchange} \right)$$

The diagram shows two incoming lines from the left meeting at a vertex, with a wavy line (representing a photon) extending to the right. At the end of the wavy line is a semi-circular grey shaded area representing a hadronic vacuum polarization insertion.

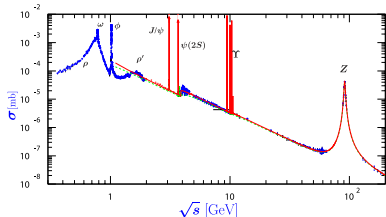
⇒ very messy!

using $t < 0$

$$\int dt \left(K'(t) \right) \left(\text{Diagram: t-channel exchange} \right)$$

The diagram shows two incoming lines from the top meeting at a vertex, with a wavy line extending downwards. At the end of the wavy line is a grey shaded circle representing a hadronic vacuum polarization insertion.

⇒ much cleaner but **smaller**



target accuracy: 10^{-5} (\rightarrow 1% on HVP)

- dominant NNLO corr. with full m dep.

$$\sigma = \int d\Phi_2 \left| \begin{array}{c} \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \end{array} \right. + \dots \left. \right|^2$$

[Carlson Calame et al. 20;
Banerjee, Engel, Signer, YU 20]

- full NNLO corr. w/o m^2/Q^2
- electronic N³LO w/o m^2/Q^2
- resummation

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$$+ \int d\Phi_3 \left| \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} + \dots \right|^2$$

$$+ \int d\Phi_4 \left| \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} + \dots \right|^2$$

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$$\begin{aligned}
 \sigma &= \int d\Phi_2 \left| \begin{array}{c} \text{tree} \\ \text{1-loop} \\ \text{2-loop} \\ \text{3-loop} \\ \dots \end{array} \right|^2 \\
 &+ \int d\Phi_3 \left| \begin{array}{c} \text{1-loop} \\ \text{2-loop} \\ \text{3-loop} \\ \dots \end{array} \right|^2 \\
 &+ \int d\Phi_4 \left| \begin{array}{c} \text{2-loop} \\ \text{3-loop} \\ \dots \end{array} \right|^2 \\
 &+ \int d\Phi_5 \left| \begin{array}{c} \text{3-loop} \\ \dots \end{array} \right|^2 \\
 &+ \text{LL} + \text{NLL} + \dots
 \end{aligned}$$

the world is not just $g - 2...$

- luminosity measurements $\Rightarrow e^+e^- \rightarrow e^+e^-$ (Belle, FCC-ee, ...)
[Banerjee, Engel, Schalch, Signer, YU 21]
- dark sector searches $\Rightarrow e^+e^- \rightarrow \gamma\gamma$ (PADME, also for luminosity...)
[Engel, Naterop, Signer, YU, Zoller 2?]
- R ratios $\Rightarrow e^+e^- \rightarrow \mu^+\mu^-$ (DAΦNE, VEPP, ...)
- τ physics $\Rightarrow e^+e^- \rightarrow \tau^+\tau^-$ (Belle)
[Kollatzsch, YU 2?]
- proton radius $\Rightarrow lp \rightarrow lp$ and $ee \rightarrow ee$ (P2, PRad, MUSE)
[Bucoveanu, Spiesberger 18; Banerjee, Engel, Signer, YU 20; Banerjee, Engel, Schalch, Signer, YU 21]
- lepton decays $\Rightarrow \ell \rightarrow \ell' \nu \bar{\nu} + \{ee, \gamma, \gamma\gamma\}$ (MEG, Mu3e, Belle, ...)
[Pruna, Signer, YU 16; YU, 17; Engel, Gnendiger, Signer, YU, 18]



MCMULE

a framework for QED corrections

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basic strategy: use 40+ years of QCD experience on QED

- use automation where available and useful (eg. OpenLoops [Buccioni, Pozzorini, Zoller 18; Buccioni, Lang, Lindert, Maierhöfer, Pozzorini 19])
- adapt QCD results where known (eg. [Bernreuther et al., 04])
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- Abelian structure \Rightarrow a bit easier [no big deal]
- much simpler infrared structure [advantage]
- want/need $m \neq 0$ since $\log m$ physical [problem]
- more exclusive, e.g. hard collinear emission [problem]

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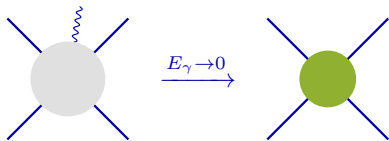
soft singularities

$$\int d\Phi_\gamma \quad \text{[Diagram: a grey circle with a wavy line and four straight lines radiating outwards]} \quad \sim \int_0^1 dE_\gamma E_\gamma \int_{-1}^1 d(\cos\theta) \frac{1}{E_\gamma^2(1 - \beta \cos\theta)}$$

soft singularities

$$\int d\Phi_\gamma \text{ (diagram with grey circle and wavy line)} \sim \int_0^1 dE_\gamma E_\gamma \int_{-1}^1 d(\cos\theta) \frac{1}{E_\gamma^2(1 - \beta \cos\theta)}$$

⇒ luckily universality of soft singularities



$$\mathcal{M}_{n+1}^{(\ell)} = \mathcal{E} \mathcal{M}_n^{(\ell)} + \text{finite}$$

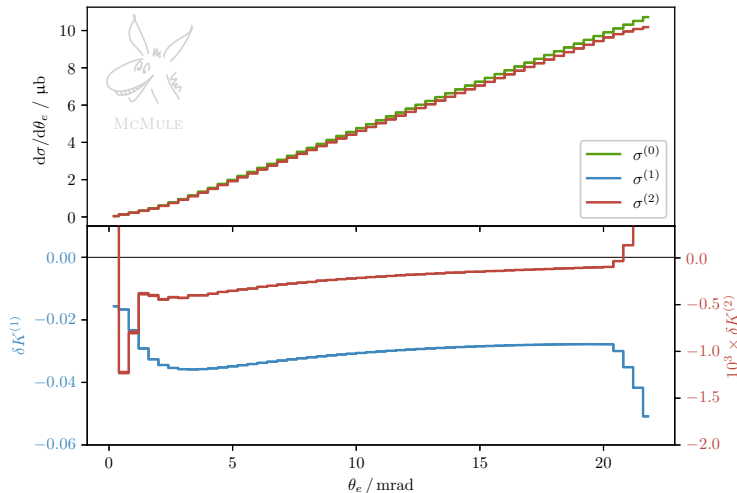
for any process and loop order

⇒ subtraction scheme at any order (FKS^ℓ) [Engel, Signer, YU 19]

$$\underbrace{\int d\Phi_\gamma}_{\text{divergent and complicated}} \text{ (grey blob)} = \underbrace{\int d\Phi_\gamma}_{\text{complicated but finite}} \left(\text{grey blob} - \text{green blob} \right) + \underbrace{\int d\Phi_\gamma}_{\text{divergent but easy}} \text{ (green blob)}$$

- very QCD-y
- based on [Frixione, Kunszt, Signer 96]
- no resolution parameter or photon mass, just DREG
- unphysical $0 < \xi_c \lesssim 1$ to test stability, implementation, ...

$E_{\mu}^{\text{beam}} = 150 \text{ GeV}$ with $E_e > 1 \text{ GeV}$, $\theta_{\mu} > 0.3 \text{ mrad}$, $0.9 < \theta_{\mu}/\theta_{\mu}^{\text{el}}(\theta_e) < 1.1$



[Banerjee, Engel, Signer, YU 20] using [Bernreuther et al. 04; Fael 18], checked [Carloni Calame et al. 20]

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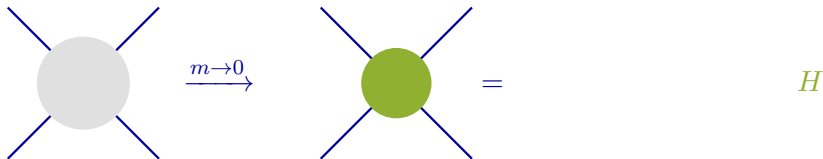
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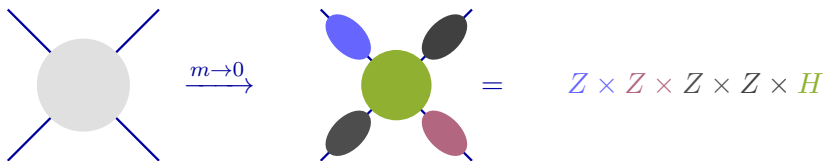
- loop integrals with internal masses are very complicated!
 - **but** $m_e^2 \ll m_\mu^2 \sim Q^2$ for many applications
- ⇒ don't actually care about full m_e dependence
- **but** $\int \langle \text{expanded integrand} \rangle \neq \langle \text{expanded integral} \rangle$
- ⇒ method of regions [Beneke, Smirnov 98] (**hard**, **soft**, **collinear**, ...)


universality of collinear singularities \rightarrow calculate up to $\mathcal{O}(m^2/Q^2)$



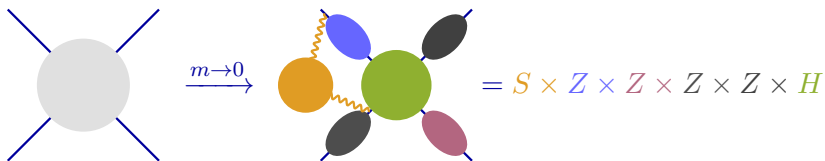
- H : hard function \sim  $|_{m=0}$


universality of collinear singularities \rightarrow calculate up to $\mathcal{O}(m^2/Q^2)$



- H : hard function \sim  $\Big|_{m=0}$
- Z : process independent jet function

universality of collinear singularities \rightarrow calculate up to $\mathcal{O}(m^2/Q^2)$



- H : hard function \sim  $\Big|_{m=0}$
- Z : process independent jet function
- S : simple soft function

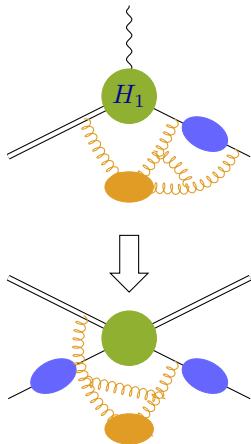
simple process ($\mu \rightarrow e\nu\nu$ or $t \rightarrow b\nu$)

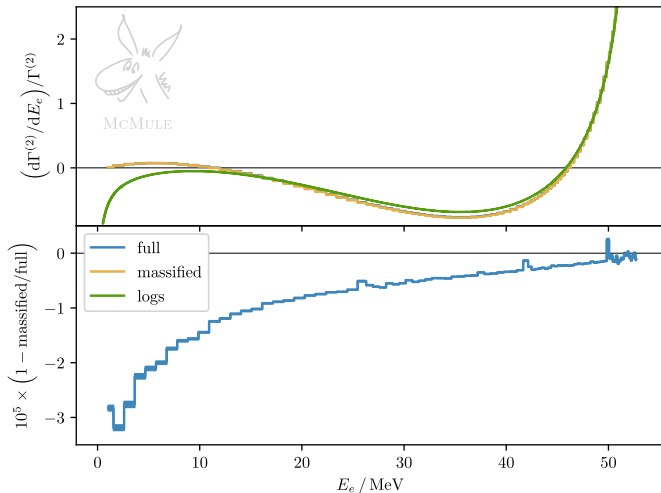
- $\mathcal{A}_\mu(m) = \mathcal{S} \times \mathcal{Z} \times \mathcal{A}_\mu(0) + \mathcal{O}(m)$
- $\mathcal{Z} \supset \log(m)$: process indep. jet fct.
- $\mathcal{S} \supset \log(m)$: process dep. soft fct. (easy)

[Penin 06; Becher, Melnikov 07; Engel, Gnendiger, Signer, YU 18]

different process ($\mu e \rightarrow \mu e$)

- $\mathcal{A}_{\mu e}(m) = \mathcal{S}' \times \mathcal{Z} \times \mathcal{Z} \times \mathcal{A}_{\mu e}(0) + \mathcal{O}(m)$





[Chen 18] v. [Engel, Gnendiger, Signer, YU 18] v. [Arbuzov et al. 02]

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real-virtual (or even real-real-virtual)

$$\mathcal{M}_{n+1}^{(\ell)} \sim \frac{1}{E_\gamma^2(1-\beta \cos \theta)}$$

- 'trivial' in principle [Buccioni, Pozzorini, Zoller 18; Buccioni, Lang, Lindert, Maierhöfer, Pozzorini et al. 19]
 - extremely delicate numerically for $E_\gamma \rightarrow 0$ (or $\cos \theta \rightarrow 1$)
- \Rightarrow Taylor expand around $E_\gamma = 0$ if small

$$\begin{aligned}
 & \text{Diagram} \xrightarrow{E_\gamma \rightarrow 0} \underbrace{\frac{1}{E_\gamma^2} \text{Diagram}}_{\text{universal eikonal}} + \underbrace{\frac{1}{E_\gamma} \text{Diagram}}_{\text{next-to-soft}} + \mathcal{O}(E_\gamma^0) \\
 & = \mathcal{E} \times \text{Diagram} + D_{\text{LBK}} \left[\text{Diagram} \right] + \left(\sum_{ijk} \mathcal{S}_{ijk} \right) \times \text{Diagram}
 \end{aligned}$$

- LBK theorem [Low 58; Burnett, Kroll 67] and extension [Engel, Signer, YU 21]

\mathcal{E} and \mathcal{S} are multiplications, what about D_{LBK} ?

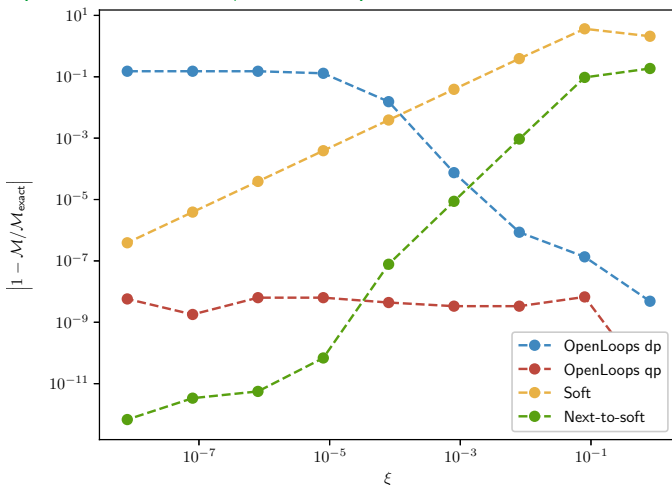
$$D_{\text{LBK}} = \frac{p_j^\mu}{k \cdot p_j} \left[\frac{p_i^\mu k^\nu}{k \cdot p_i} - g^{\mu\nu} \right] \frac{\partial}{\partial p_i^\nu}$$

- have to take derivative wrt. external momenta
- usually done analytically
- can be done numerically as well using phase space routine
 PS : $\vec{x} \rightarrow \{p_i\}$ using

$$\frac{\partial}{\partial p_i^\nu} = \frac{\partial s_{jk}}{\partial p_i^\nu} \left[\frac{\partial s_{jk}}{\partial x_l} \right]^{-1} \frac{\partial}{\partial x_l}$$

- naive numerical derivative ($h = 10^{-8}$) in MCMULE

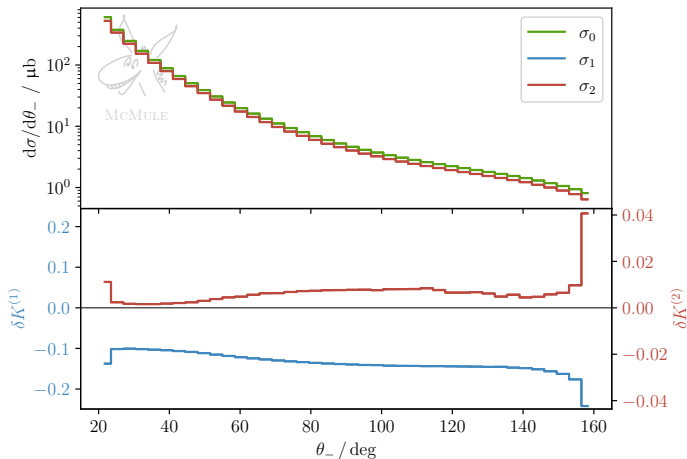
example $e^+e^- \rightarrow e^+e^-\gamma$ @ one-loop



compare with exact calculation in Mathematica

[Banerjee, Engel, Schalch, Signer, YU 21]

$\sqrt{s} = 1020 \text{ MeV}$



$E_{\pm} > 408 \text{ MeV}, 20^\circ \leq \theta_{\pm} \leq 160^\circ, |180^\circ - \theta_+ - \theta_-| < 10^\circ$

visions for the future

- polarisation effects ($ee \rightarrow \tau\tau$, $lp \rightarrow lp$, ...)
- EW effects ($ee \rightarrow \tau\tau$, $lp \rightarrow lp$, $ee \rightarrow ee$, $ee \rightarrow \gamma\gamma$, ...)
- finish NNLO for μ - e scattering
- HLbL in $ee \rightarrow \gamma\gamma$ & TPE in $lp \rightarrow lp$
- higher orders (N³LO for $e\mu \rightarrow e\mu$ or NNLO for $ee \rightarrow \mu\mu\gamma$)
- resummation using parton shower
- improve / extend NTS (adding to OpenLoops w/ QCD results?) [Laenen et al. 17, 19, 20]

10^{-5} is really small... could we go to N³LO?

⇒ quite possibly (for the dominant contributions)!

- three-loop heavy-quark form factor: known semi-analytically [Fael, Lange, Schönwald, Steinhauser 22]
- real-virtual-virtual: known for $m_e = 0$ [Gehrmann, Remiddie 01]

⇒ massification to the rescue!

- real-real-virtual: OpenLoops \oplus NTS [Buccioni, Pozzorini, Zoller 18; Buccioni, Lang, Lindert, Maierhöfer, Pozzorini 19; Banerjee, Engel, Schalch, Signer, YU 21; Engel, Signer, YU 21]
- real-real-real: 'trivial'

kick-off workstop in summer at IPPP!



MCMULE

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