2nd workstop

"Theory for muon-electron scattering @ 10ppm"

From matrix elements to a NNLO parton-level Monte Carlo for μ-e scattering

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for S4

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5TH February 2019

the problem of precision calculation

- loop integrals become harder the more masses are introduced $\to$ treat fermions massless wherever possible
- phase-space integrals become easier when more masses are introduced $\to$ keep fermion masses wherever possible
- fermions have masses but with large separation

use this to our advantage

- calculate the two-loop integrals without masses
- do some magic (aka. SCET-ish) to obtain small-mass expansion of two-loop matrix element
- calculate phase-space integrals numerically with $m > 0$ exactly
- result is correct up to $\mathcal{O}(z)$

landscape

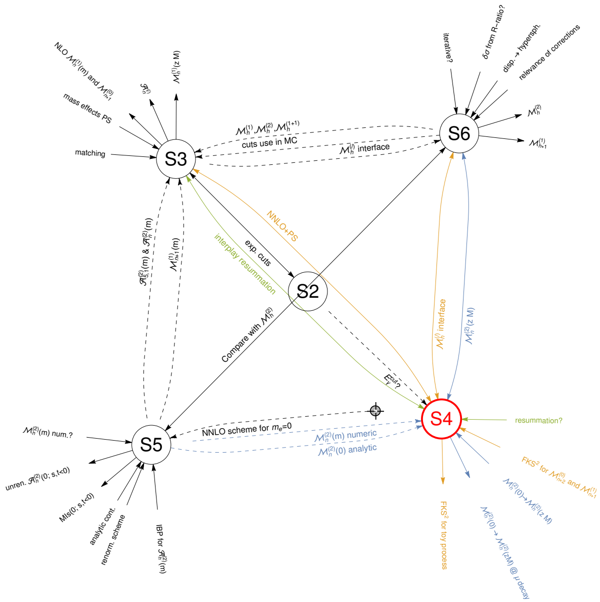
we assume:

- LO and NLO calculation w/ full m dependence
- $\mathcal{M}_n^{(2)}(0)$ is known (see S5 tomorrow)
- $\mathcal{M}_n^{(2)}(m)$ may be possible numerically (cross check)

we want to provide:

- fully differential MC up to NNLO
- drop terms suppressed by $\alpha^2 z$ (rel. to LO)
- keep αz^n and $\alpha^2(\ln z)^n$

landscape


S2
S3
S4
S5
S6
NLO $\mathcal{M}_n^{(1)}$(m) and $\mathcal{M}_{n+1}^{(0)}$
mass effects PS
matching
$\mathcal{M}_n^{(1)}$(z M)
$\mathcal{M}_n^{(1)}$,$\mathcal{M}_n^{(2)}$,$\mathcal{M}_n^{(1+1)}$
cuts use in MC
$\mathcal{M}_n^{(l)}$ interface
iterative?
δσ from R-ratio?
disp. → hypersph.
relevance of corrections
$\mathcal{M}_n^{(2)}$
$\mathcal{M}_{n+1}^{(1)}$
NNLO+PS
exp. cuts
interplay resummation
$\mathcal{A}_{5,1}^{(2)}$(m) & $\mathcal{A}_n^{(2)}$(m)
$\mathcal{M}_{n+1}^{(1)}$(m)
Compare with $\mathcal{M}_n^{(2)}$
$\mathcal{M}_n^{(l)}$ interface
$\mathcal{M}_n^{(2)}$(z M)
E_γ^{cut}?
NNLO scheme for m_e=0
$\mathcal{M}_n^{(2)}$(m) numeric
$\mathcal{M}_n^{(2)}$(0) analytic
resummation?
FKS² for $\mathcal{M}_{n+2}^{(0)}$ and $\mathcal{M}_{n+1}^{(1)}$
$\mathcal{M}_n^{(2)}$(0)→$\mathcal{M}_n^{(2)}$(z M)
$\mathcal{M}_n^{(2)}$(0) → $\mathcal{M}_n^{(2)}$(zM) @ μ decay
FKS² for toy process
$\mathcal{M}_n^{(2)}$(m) num.?
unren. $\mathcal{A}_n^{(2)}$(0; s,t<0)
MIs(0; s,t<0)
analytic cont.
renorm. scheme
IBP for $\mathcal{A}_n^{(2)}$(m)

“massification” of $\mathcal{M}_n^{(2)}$

$m = 0 \;\rightarrow\; m \neq 0$

Use SCET inspired way to relate $m = 0 \rightarrow \; m \neq 0$ [Becher, Melnikov 07]

($\sim$ fragmentation function approach)

Form factor: (only one external mass $m \ll Q^2 = s$)

$$F(s,m) = Z_J(m^2)\, S(s,m)\, \tilde{F}(s) + \mathcal{O}(m^2/s)$$

- $S(s,m)$: soft function, only contributions from vacuum polarization diagrams with massive fermions, $\supset \ln(m^2/s)$
- $Z_J(m^2)$: jet fct., independent of hard scale s, $\supset \ln(m^2/m_i^2)$
- $\tilde{F}(s)$: massless form factor
- factorisation $\leftrightarrow$ resummation via RG equations

used for Bhabha scattering (one mass $m \ll s,t,u$) [Becher, Melnikov 07]

$m = 0 \;\rightarrow\; m \neq 0$

- extend this to processes with two external masses, M and m (in QCD)
- calculate simplest example $t(p) \to b(q) + W^\pm$ and compare with $\gamma^* \to q\bar{q}$ (replace t with μ etc)
- identify momentum regions h, c, s and us (drops out)
- reduction mixes soft/collinear regions confusingly $\Rightarrow$ do this at the diagram level
- $F_{t\to bW^\pm}(zM) = \mathcal{S} \times \sqrt{Z_q} \times F_{t\to bW^\pm}(0)$
  $F_{\gamma^*\to q\bar{q}}(zM) = \mathcal{S}' \times \sqrt{Z_q} \times \sqrt{\bar{Z}_q} \times F_{\gamma^*\to q\bar{q}}(0)$
- μ–e scattering and $\gamma^* \to q\bar{q}$ have $\bar{c}$ regions

example: n_m contribution

- soft

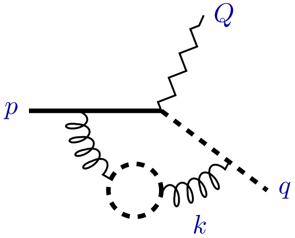
$$\propto \int \frac{\Pi^{(n_m)}(k)}{(k^2)^2(2p\cdot k)(2q_-\cdot k)} = \infty$$

- factorisation anomaly!

$\Rightarrow$ analytic regularisation

$$\int \frac{\Pi^{(n_m)}(k)}{(k^2)^2(2p\cdot k)^{1+\eta}(2q_-\cdot k)} \propto \frac{m^{-\eta}}{\eta}$$

- $Z_q \supset -\frac{s^\eta}{\eta}$, inducing $\log\frac{s}{m}$
- new feature for $m \ll M$
- in $\gamma^* \to q\bar q$ cancelled in product $\sqrt{\bar Z_q}\times\sqrt{Z_q}$



summary: part I

- collinear contributions $\sqrt{Z_q}$ and $\sqrt{\bar{Z}_q}$ known
- what are the induced logs?
- loop corrections break scaling symmetry of SCET, introducing factorisation anomaly
- we need $\mathcal{O}(\epsilon^2)$ of $\mathcal{M}_n^{(1)}(0)$, $\mathcal{M}_n^{(2)}(0)$ and the process dependent soft function
- error due to the expansion $\alpha^2\mathcal{O}(10^{-3})$

FKS^2: double-soft extension of FKS

real correction at NLO

The FKS formalism at NLO

- no collinear singularities in μ–e $\to$ very simple scheme
- everything that `vegas` sees needs to be finite!
- let $E_\gamma \propto \xi$, introduce arbitrary $0 < \xi_\text{cut} \le 1$

$$\underbrace{\mathrm{d}\phi_{n+1}}_{\propto \xi^{1-2\epsilon}\mathrm{d}\xi}\ \underbrace{\mathcal{M}^{(0)}_{n+1}}_{\supset \xi^{-2}} \propto \mathrm{d}\phi_n \times \mathrm{d}\xi\,\mathrm{d}\Omega\ \xi^{-1-2\epsilon}\underbrace{\Big(\xi^2\mathcal{M}^{(0)}_{n+1}\Big)}_{\text{reg. }\xi\to 0}$$

$$\propto \mathrm{d}\Omega\Bigg(\underbrace{-\frac{\xi_\text{cut}{}^{-2\epsilon}}{2\epsilon}\delta(\xi)}_{(s)} + \underbrace{\big(\xi^{-1-2\epsilon}\big)_{\xi_\text{cut}}}_{(h)}\Bigg)\Big(\xi^2\mathcal{M}^{(0)}_{n+1}\Big)$$

with $\int \mathrm{d}\xi\,(\xi^n)_{\xi_\text{cut}} f(\xi) = \int \mathrm{d}\xi\,\xi^n\Big(f(\xi) - f(0)\theta(\xi_\text{cut}-\xi)\Big)$

real correction at NLO

- $\mathrm{d}\sigma^{(h)}$ is finite $\Rightarrow$ $\epsilon \to 0$ numerically
- $\mathrm{d}\sigma^{(s)}$ is 'trivial' because of $\delta(\xi)$

$$
\begin{aligned}
\mathrm{d}\sigma^{(s)} &\propto \frac{{\xi_{\text{cut}}}^{-2\epsilon}}{2\epsilon} \int \mathrm{d}\Omega \, \Big(\xi^2 \mathcal{M}_{n+1}^{(0)}\Big)_{\xi=0} \\
&\propto \frac{{\xi_{\text{cut}}}^{-2\epsilon}}{2\epsilon} \mathcal{M}_n^{(0)} \int \mathrm{d}\Omega \, \mathcal{E} \\
&= \hat{\mathcal{E}}(\xi_{\text{cut}}) \mathcal{M}_n^{(0)}
\end{aligned}
$$

- eikonal $\mathcal{E}$ is build from building blocks
- $\hat{\mathcal{E}} \mathcal{M}_n^{(0)} + \mathcal{M}_n^{(1)} = \text{finite}$ (KLN)
- use ξ_{cut} to test implementation (any IR safe $\frac{\mathrm{d}^n\sigma}{\mathrm{d}x_1 \cdots \mathrm{d}x_n}$ can't depend on ξ_{cut})

FKS extension to NNLO: FKS^2

term 1: $\mathcal{M}^{(1)}_{n+1}$ (real$\times$virtual)

- like NLO, but $\mathrm{d}\sigma^{(h)} \propto \mathcal{M}^{(1)}_{n+1}$ is not finite $\Rightarrow$ split further
  1. chop the pole (and induced terms): $\overline{\text{MS}}$-like subtraction
  2. $\mathcal{M}^{(1)}_{n+1} = \underbrace{\mathcal{M}^{(1)f}_{n+1}}_{\Rightarrow \mathrm{d}\sigma^{(fin)}} \underbrace{-\hat{\mathcal{E}}(\xi^2_{\text{cut}})\mathcal{M}^{(0)}_{n+1}}_{\Rightarrow \mathrm{d}\sigma^{(sin)}}$: eikonal subtraction

  $\mathcal{M}^{(1)f}_{n+1}$ is finite (KLN of the $n+1$ process)

- $\mathrm{d}\sigma^{(fin)}$ and $\mathrm{d}\sigma^{(sin)}$ depend on two a-priori different ξ^i_{cut}
- $\mathrm{d}\sigma^{(fin)}$ is now finite $\Rightarrow$ $\epsilon \to 0$
- we postpone

$$\mathrm{d}\sigma^{(sin)} \equiv -\mathcal{I}(\xi^1_{\text{cut}}, \xi^2_{\text{cut}}) \propto \int_{\xi^1_{\text{cut}}} \hat{\mathcal{E}}(\xi^2_{\text{cut}})\ (\xi^2 \mathcal{M}^{(0)}_{n+1})$$

FKS extension to NNLO: FKS^2

term 2: $\mathcal{M}^{(0)}_{n+2}$ (real $\times$ real)

- do normal FKS twice with two ξ_{cut}, but (h) mixes with (s)

$$\mathrm{d}\sigma_{n+2} = \underbrace{\mathrm{d}\sigma^{(hh)}}_{\epsilon\to 0} + \underbrace{\mathrm{d}\sigma^{(ss)}}_{\hat{\mathcal{E}}(\xi^A_{\text{cut}})\,\hat{\mathcal{E}}(\xi^B_{\text{cut}})} + \mathrm{d}\sigma^{(hs)} + \mathrm{d}\sigma^{(sh)}$$

- mixed terms $\mathrm{d}\sigma^{(hs)}$ are troublesome

$$\mathrm{d}\sigma^{(hs)} \propto \frac{1}{2!}\int_{\xi^A_{\text{cut}}} \hat{\mathcal{E}}(\xi^B_{\text{cut}})\ (\xi^2\mathcal{M}^{(0)}_{n+1}) = \frac{1}{2!}\mathcal{I}(\xi^A_{\text{cut}}, \xi^B_{\text{cut}})$$

- luckily $\mathrm{d}\sigma^{(aux)} \equiv \mathrm{d}\sigma^{(sin)} + \mathrm{d}\sigma^{(hs)} + \mathrm{d}\sigma^{(sh)} = 0$ if all ξ^i_{cut} are equal
- the sum $\mathrm{d}\sigma^{(ss)} + \mathrm{d}\sigma^{(s)} + \mathrm{d}\sigma^{(aux)} + \mathrm{d}\sigma_{VV}(m)$ is finite (KLN)

summary part II

assumptions

- $\mathcal{M}_n^{(1)}(m)$ is known up to $\mathcal{O}(\epsilon^1)$
  $\Rightarrow$ annoying but not a real bottle-neck
- poles of $\mathcal{M}_n^{(2)}(m)$ known exactly (prediction from SCET)
- finite part of $\mathcal{M}_n^{(2)}(m)$ or $\mathcal{M}_n^{(2)}(zM)$ introducing an error $\mathcal{O}(m/\{s,t,M\})$
- $\mathcal{M}_{n+1}^{(1)}(m)$ can be implemented in `FORTRAN` numerically stable

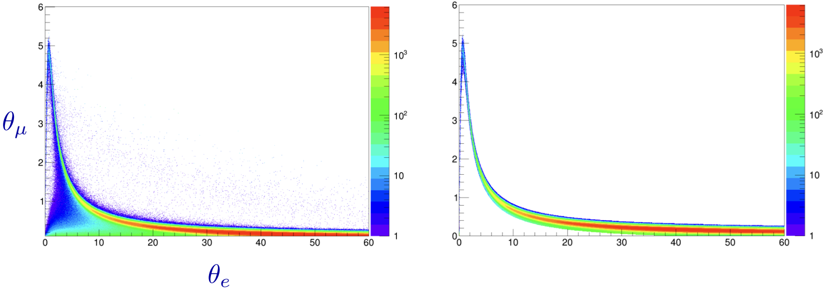
scheme tested for $e^+e^- \to \nu\bar{\nu}$

next: test for μ decay

Resummation

a few comments

- resumming $\log m$ by Z_q
- resumming cuts $\Rightarrow$ simple, not too stringent



Open Questions

next step for us

calculate $\propto q^4(q\,Q)^2$ MC integrator

- rotate toy process $e^+e^- \to \nu\bar\nu$
- $\mathcal{A}_{5,1}^{(2)}(m)$ from [Bernreuther et al. 04]
- $\mathcal{A}_{4,1}^{(1)} \times \mathcal{A}_{2,1}^{(0)}$ with full m dependence (x-check GoSam?)
- result has full m dependence
- tests massification
- compare S3's PS
- ep scattering with $m_\gamma > 0$ [Bucoveanu, Spiesberger 18]

open questions

$\mathcal{M}_n^{(2)}(zM)$

- we need $\mathcal{M}_n^{(2)}(0)$ from S5
- can we cross check our expansion in a sensible way, S5?
- we need to compute the soft function $\mathcal{S}$

fixed order MC

- we need $\mathcal{M}_{n+1}^{(1)}(m)$. anybody fancy doing that?
- what cuts do we want? (not important just yet)
- can we actually implement this without messing up?

resummation

- what needs to be resummed? $\log m$? $\log E_\gamma^{\text{cut}}$?
- how would we go about resumming $\log E_\gamma^{\text{cut}}$?

things I didn't mention...

... mostly because they aren't worked out yet

- a whole lot of details
- hadronic contributions
- can we cross check our Monte Carlo using reverse unitarity?
- can we magic up the $\mathcal{O}(z)$ terms?